Challenges in Privacy-Preserving Analysis of Structured Data

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Sensitive Structured Data

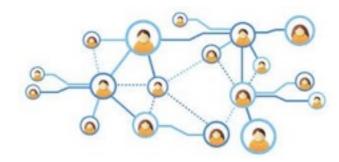
Medical Records



Search Logs



Social Networks

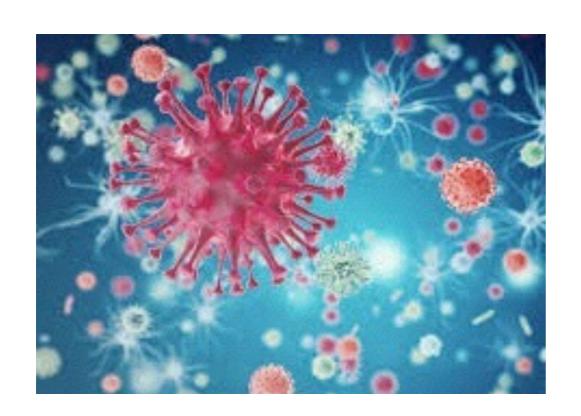


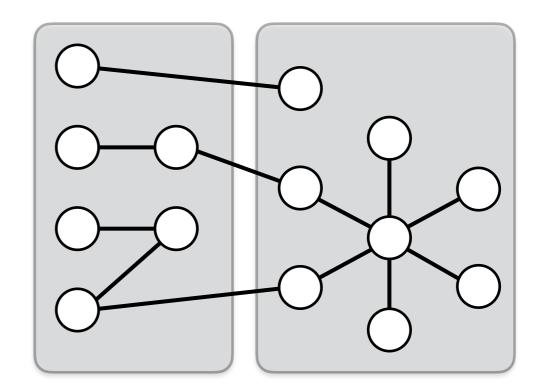
This Talk: Two Case Studies

I. Privacy-preserving HIV Epidemiology

2. Privacy in Time-series data

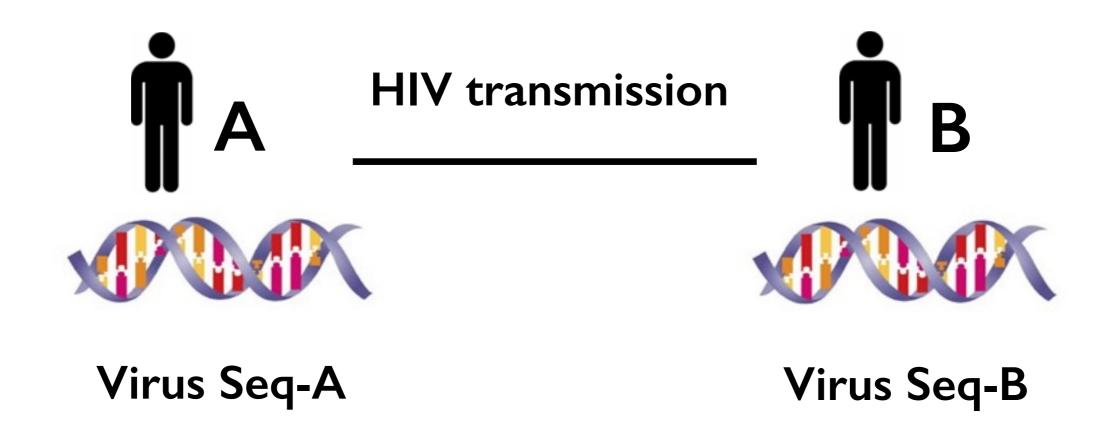
HIV Epidemiology





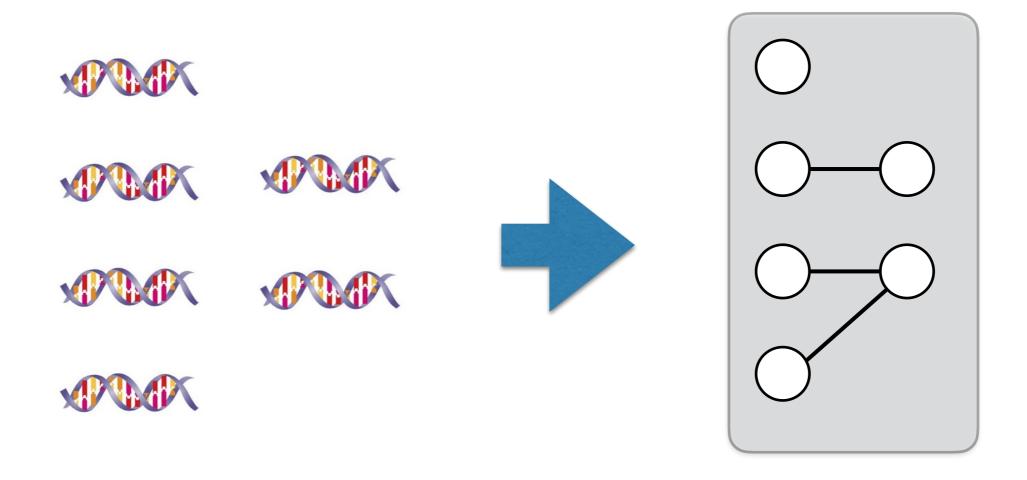
Goal: Understand how HIV spreads among people

HIV Transmission Data



distance (Seq-A, Seq-B) < t

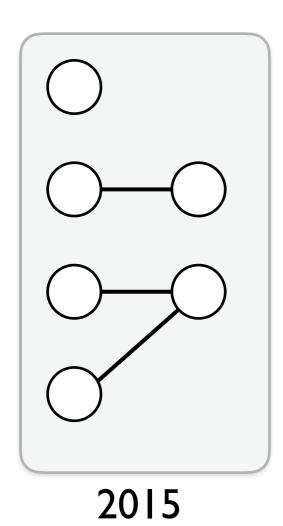
From Sequences to Transmission Graphs



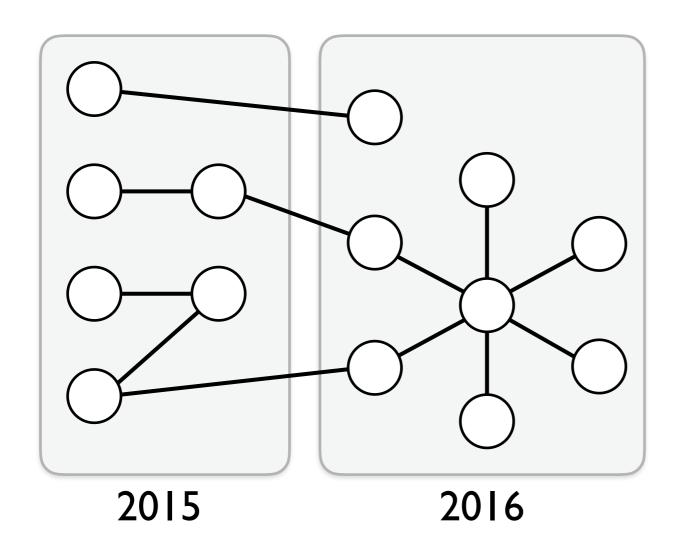
Viral Sequences

Node = Patient Edge = Plausible

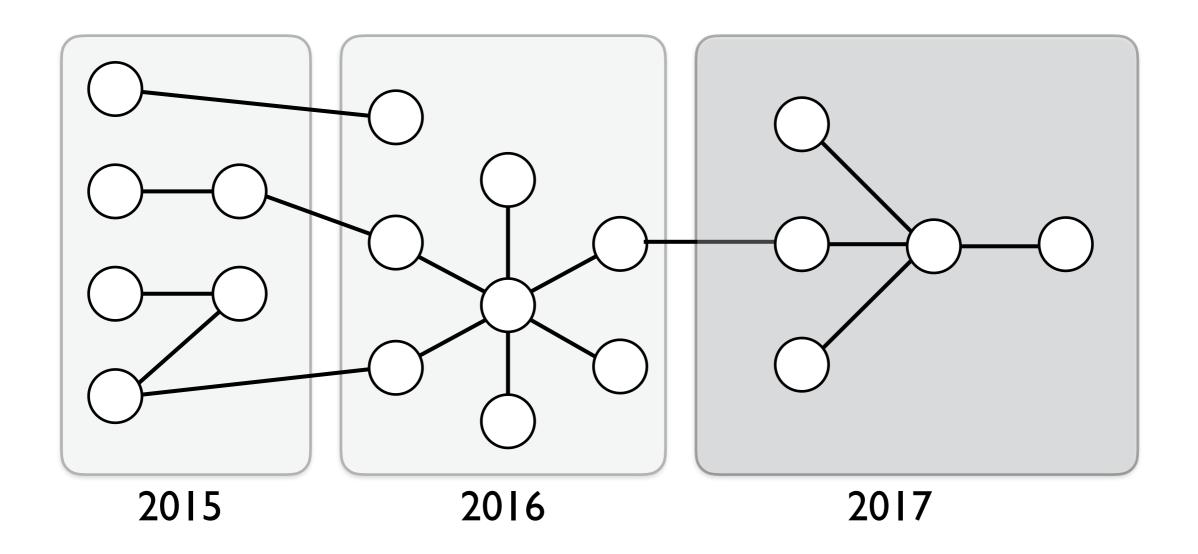
transmission



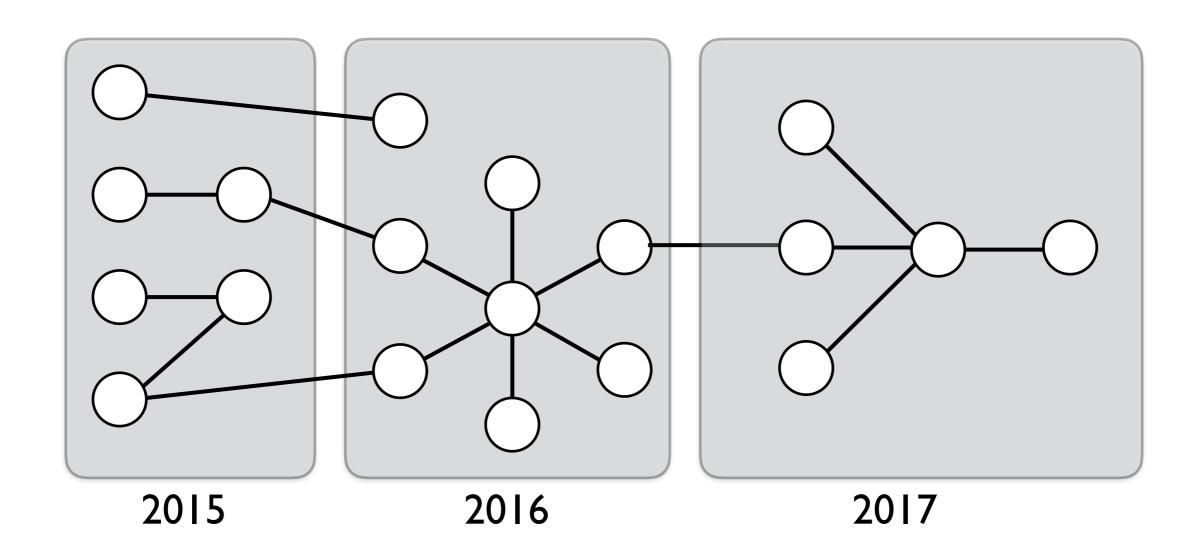
Node = Patient



Node = Patient

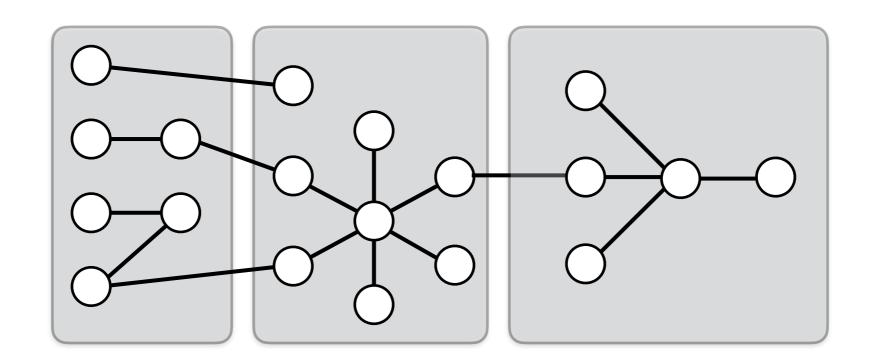


Node = Patient



Goal: Release properties of G with privacy across time

Problem: Continual Graph Statistics Release



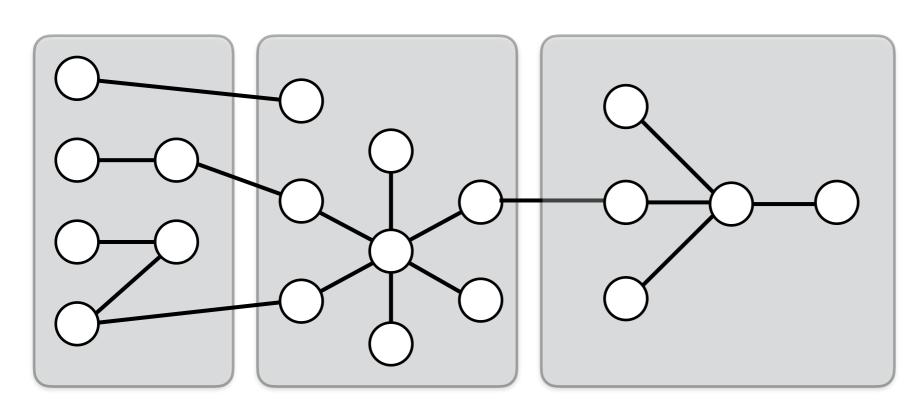
Given: (Growing) graph G

At time t, nodes and adjacent edges $(\partial V_t, \partial E_t)$ arrive

Goal: At time t, release $f(G_t)$, where f = graph statistic, and $G_t = (\bigcup_{s \le t} \partial V_s, \bigcup_{s \le t} \partial E_s)$

while preserving patient privacy and high accuracy

What kind of Privacy?



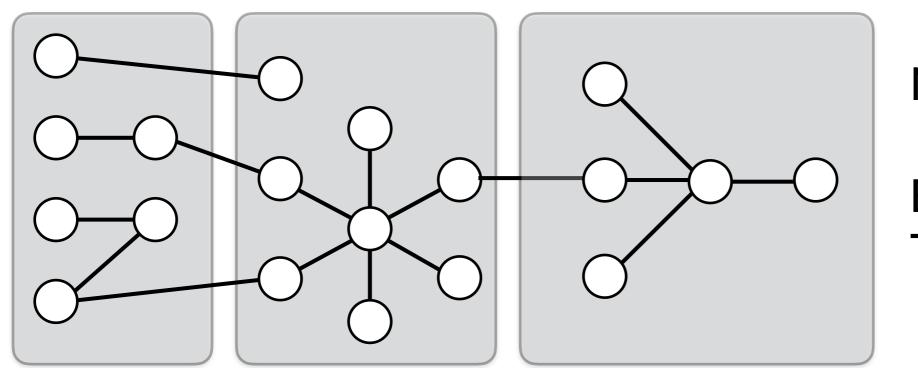
Node = Patient

Edge = Transmission

Hide: Patient A is in the graph

Release: Large scale properties

What kind of Privacy?



Node = Patient

Edge = Transmission

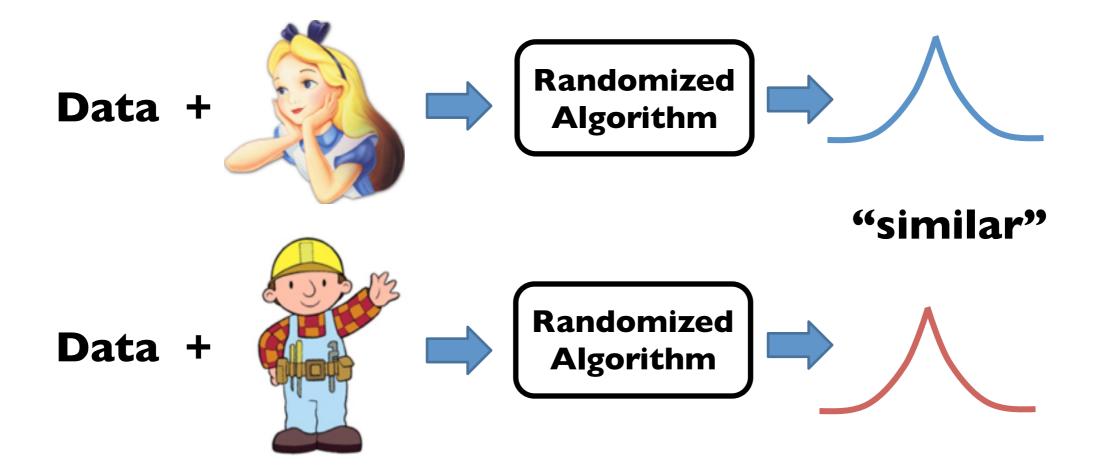
Hide. A particular patient has HIV

Privacy notion: Node Differential Privacy

Talk Outline

- The Problem: Private HIV Epidemiology
- Privacy Definition: Differential Privacy

Differential Privacy [DMNS06]



Participation of a single person does not change output

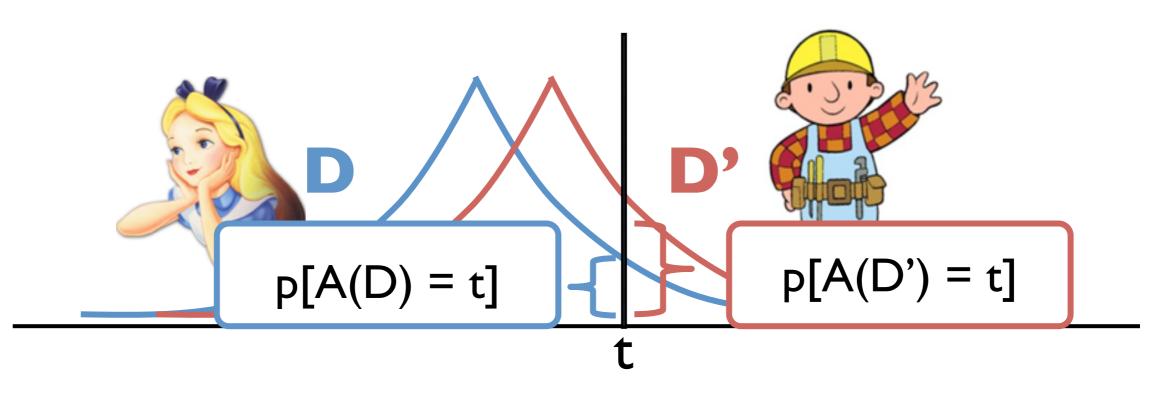
Differential Privacy: Attacker's View





- Note: a. Algorithm could draw personal conclusions about Alice
 - b. Alice has the agency to participate or not

Differential Privacy [DMNS06]



For all D, D' that differ in one person's value, If $A = \epsilon$ -differentially private randomized algorithm, then:

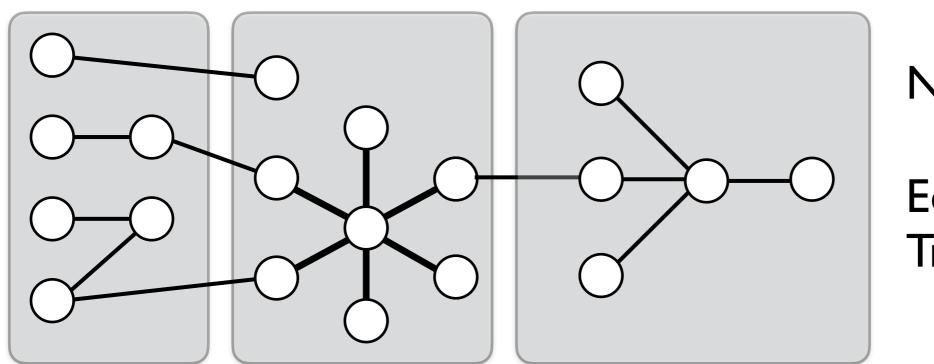
$$\sup_{t} \left| \log \frac{p(A(D) = t)}{p(A(D') = t)} \right| \le \epsilon$$

Differential Privacy

I. Provably strong notion of privacy

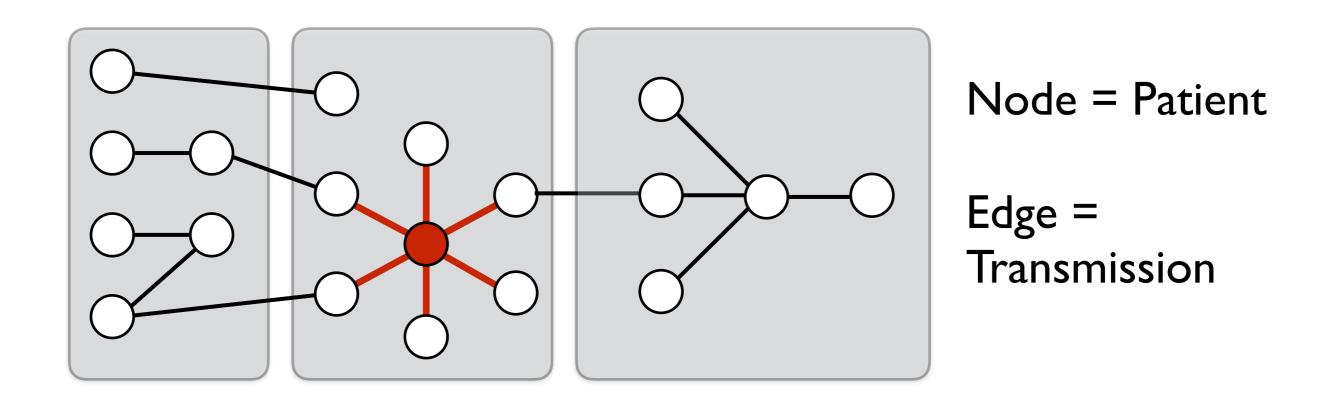
2. Good approximations for many functions e.g, means, histograms, etc.

Node Differential Privacy



Node = Patient

Node Differential Privacy

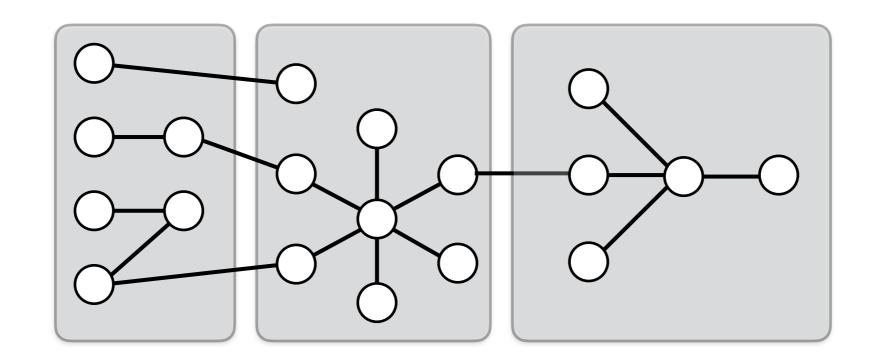


One person's value = One node + adjacent edges

Talk Outline

- The Problem: Private HIV Epidemiology
- Privacy Definition: Node Differential Privacy
- Challenges

Problem: Continual Graph Statistics Release



Given: (Growing) graph G

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Goal: At time t, release $f(G_t)$, where f = graph statistic, and $G_t = (\bigcup_{s \le t} \partial V_s, \bigcup_{s \le t} \partial E_s)$

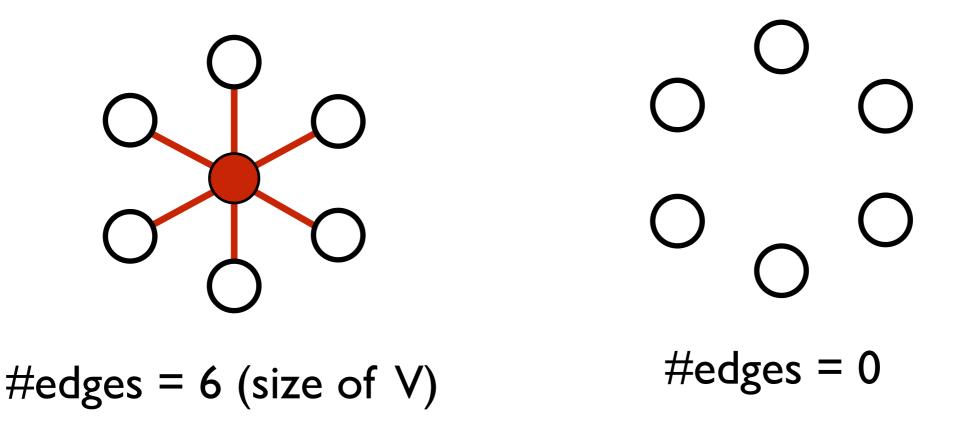
with node differential privacy and high accuracy

Why is Continual Release of Graphs with Node Differential Privacy hard?

- I. Node DP challenging in static graphs [KNRS13, BBDS13]
- 2. Continual release of graph data has extra challenges

Challenge I: Node DP

Removing one node can change properties by a lot (even for static graphs)



Hiding one node needs high noise — low accuracy

Prior Work: Node DP in Static Graphs

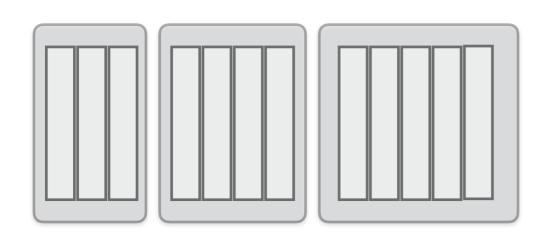
Approach I [BCS15]:

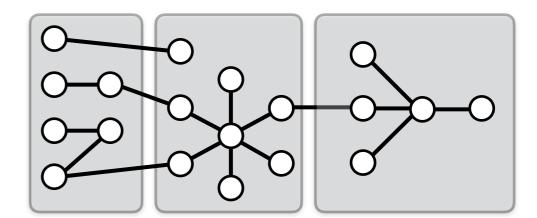
- Assume bounded max degree

Approach 2 [KNRS13, RS15]:

- Project to low degree graph G' and use node DP on G'
- Projection algorithm needs to be "smooth" and computationally efficient

Challenge 2: Continual Release of Graphs



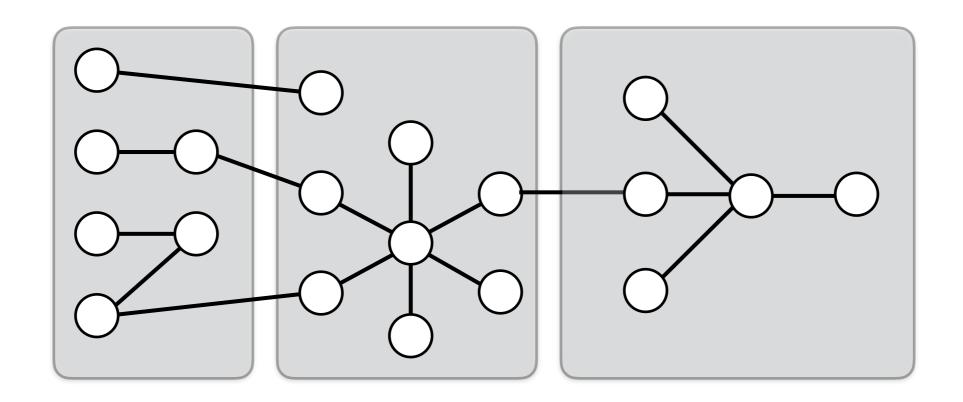


- Methods for tabular data [DNPR10, CSS10] do not apply
- Sequential composition gives poor utility
- Graph projection methods are not "smooth" over time

Talk Outline

- The Problem: Private HIV Epidemiology
- Privacy Definition: Node Differential Privacy
- Challenges
- Approach

Algorithm: Main Ideas

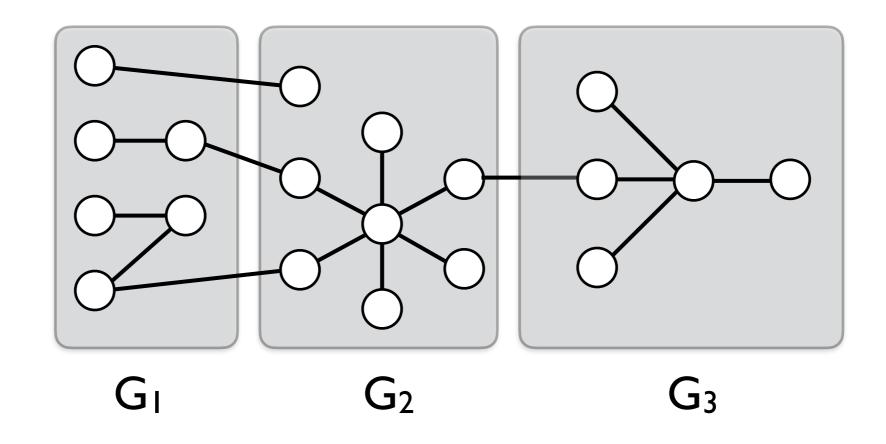


Strategy I: Assume bounded max degree of G (from domain)

Strategy 2: Privately release "difference sequence" of statistic (instead of the direct statistic)

Difference Sequence

Graph Sequence:



Statistic Sequence:

 $f(G_I)$

 $f(G_2)$

 $f(G_3)$

Difference Sequence:

 $f(G_1)$

 $f(G_2) - f(G_1)$ $f(G_3) - f(G_2)$

Key Observation

Key Observation: For many graph statistics, when G is degree bounded, the difference sequence has low sensitivity

Example Theorem:

If max degree(G) = D, then sensitivity of the difference sequence for #high degree nodes is at most 2D + I.

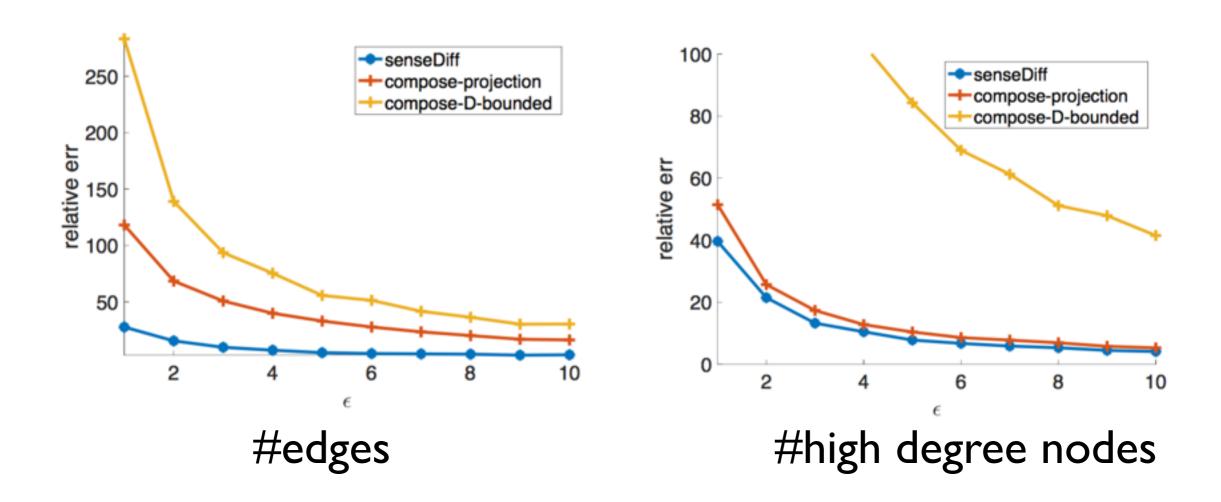
From Observation to Algorithm

Algorithm:

- I. Add noise to each item of difference sequence to hide effect of single node and publish
- 2. Reconstruct private statistic sequence from private difference sequence

How does this work?

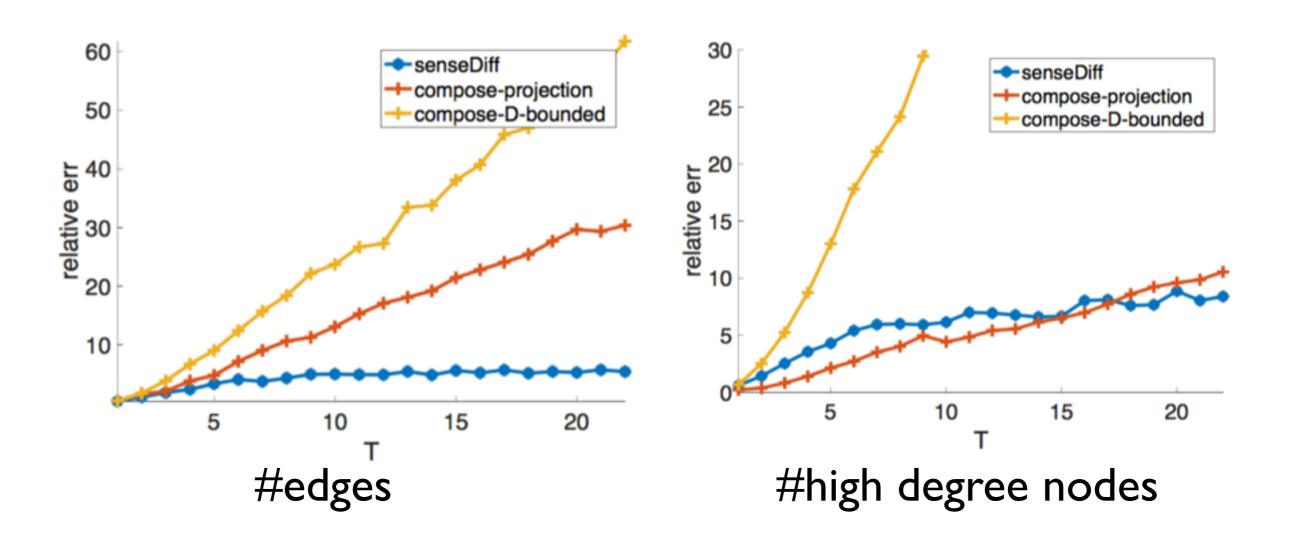
Experiments - Privacy vs. Utility



Baselines:

Our Algorithm, DP Composition 1, DP Composition 2

Experiments - #Releases vs. Utility



Baselines:

Our Algorithm, DP Composition 1, DP Composition 2

Talk Agenda

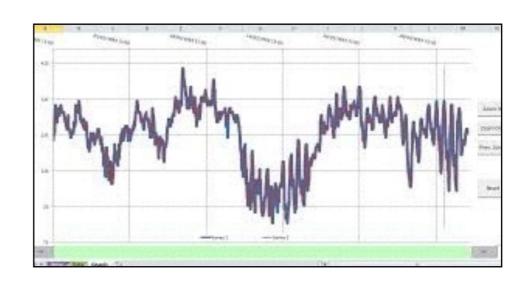
Privacy is application-dependent!

Two applications:

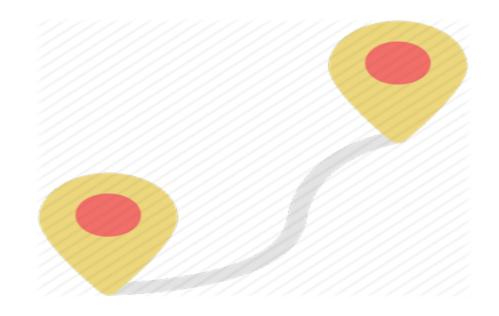
- I. HIV Epidemiology
- 2. Privacy of time-series data activity monitoring, power consumption, etc

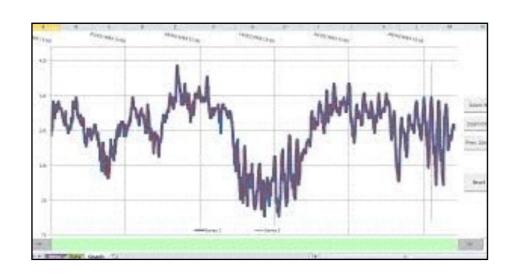
Time Series Data

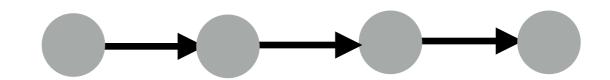
Physical Activity
Monitoring



Location traces







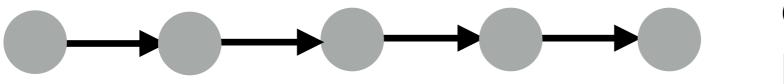
Data: Activity trace of a subject

Hide: Activity at each time against adversary with prior knowledge

Release: (Approximate) aggregate activity

Why is Differential Privacy not Right for Correlated data?

$$D = (x_1, ..., x_T), x_t = activity at time t$$



Correlation Network

Data from a single subject

I-DP: Output histogram of activities + noise with stdev T

Too much noise - no utility!

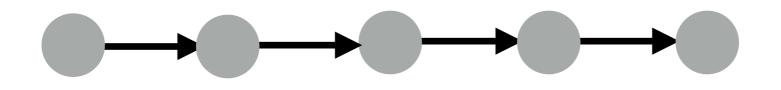
$$D = (x_1, ..., x_T), x_t = activity at time t$$



Correlation Network

I-entry-DP: Output activity histogram + noise with stdev I Not enough noise - activities across time are correlated!

$$D = (x_1, ..., x_T), x_t = activity at time t$$

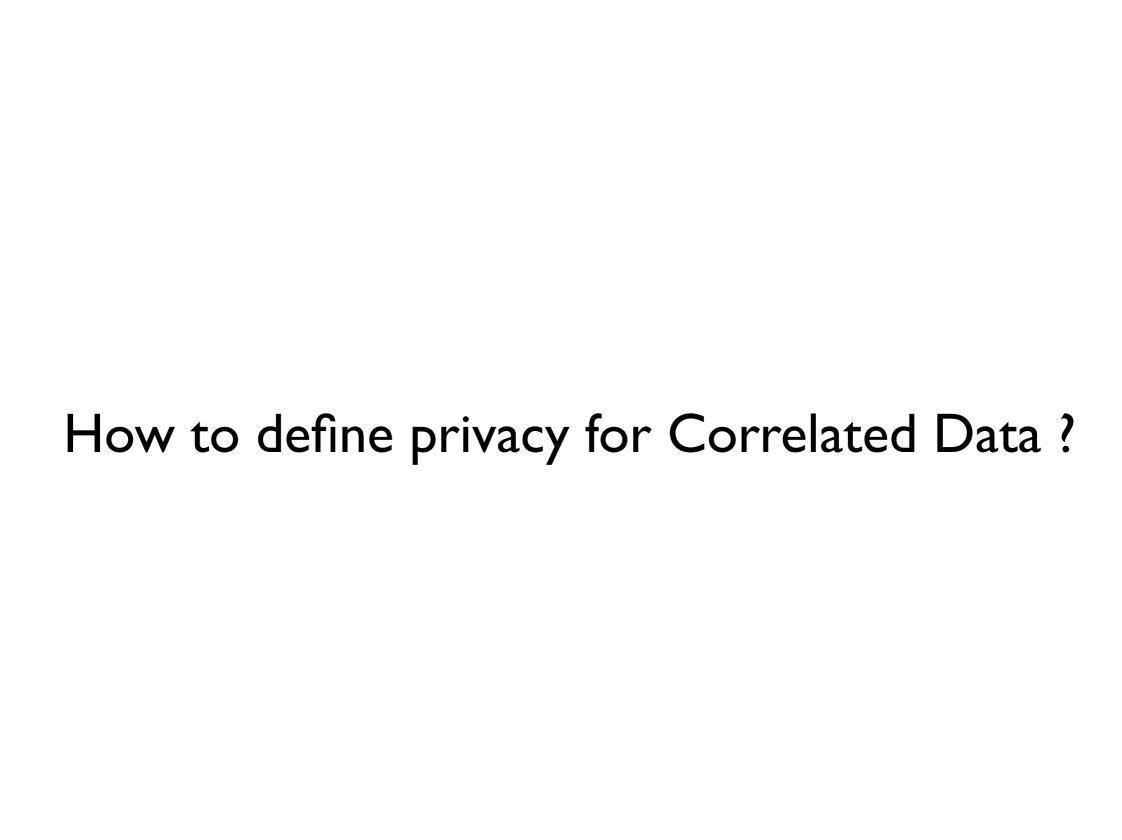


Correlation Network

I-entry-group DP:

Output activity histogram + noise with stdev T

Too much noise - no utility!



Secret Set S

S: Information to be protected

e.g: Alice's age is 25, Bob has a disease

Secret Set S

Secret Pairs
Set Q

Q: Pairs of secrets we want to be indistinguishable

e.g: (Alice's age is 25, Alice's age is 40)

(Bob is in dataset, Bob is not in dataset)

Secret Set S

Secret Pairs
Set Q

Distribution Class (-)

 Θ : A set of distributions that plausibly generate the data

e.g: (connection graph G, disease transmits w.p [0.1, 0.5]) (Markov Chain with transition matrix in set **P**)

May be used to model correlation in data

Secret Set S

Secret Pairs
Set Q

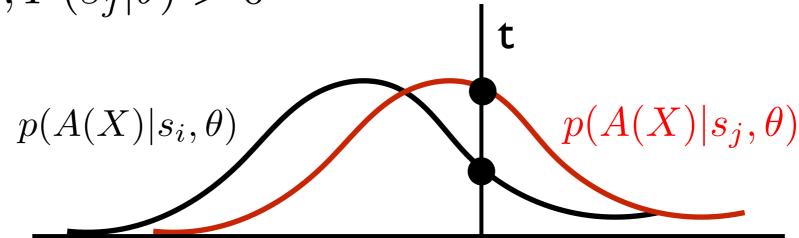
Distribution Class (-)

An algorithm A is ϵ -Pufferfish private with parameters

 (S,Q,Θ) if for all (s_i, s_j) in Q, for all $\theta \in \Theta$, $X \sim \theta$, all t,

$$p_{\theta,A}(A(X) = t|s_i, \theta) \le e^{\epsilon} \cdot p_{\theta,A}(A(X) = t|s_j, \theta)$$

whenever $P(s_i|\theta), P(s_j|\theta) > 0$



Pufferfish Interpretation of DP

```
Theorem: Pufferfish = Differential Privacy when:

S = \{ s_{i,a} := \text{Person i has value a, for all i, all a in domain } X \}

Q = \{ (s_{i,a} s_{i,b}), \text{ for all i and (a, b) pairs in } X \times X \}

\Theta = \{ \text{ Distributions where each person i is independent } \}
```

Pufferfish Interpretation of DP

Theorem: Pufferfish = Differential Privacy when: $S = \{ s_{i,a} := Person i has value a, for all i, all a in domain X \}$

 $Q = \{ (s_{i,a} s_{i,b}), \text{ for all } i \text{ and } (a,b) \text{ pairs in } X \times X \}$

 $\Theta = \{ \text{ Distributions where each person i is independent } \}$

Theorem: No utility possible when:

 $\Theta = \{ All possible distributions \}$

How to get Pufferfish privacy?

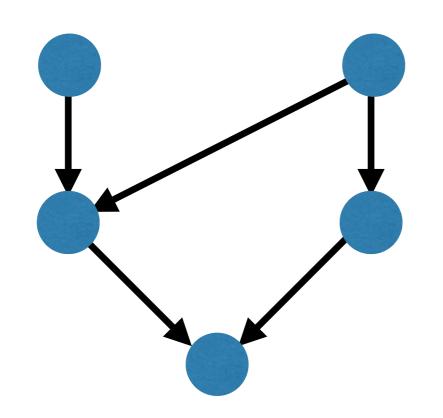
Special case mechanisms [KM12, HMD12]

Is there a more general Pufferfish mechanism for a large class of correlated data?

Our work: Yes, the Markov Quilt Mechanism

(Also concurrent work [GK16])

Correlation Measure: Bayesian Networks



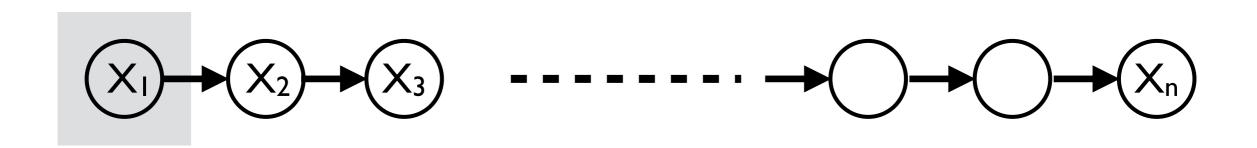
Node: variable

Directed Acyclic Graph

Joint distribution of variables:

$$\Pr(X_1, X_2, \dots, X_n) = \prod_i \Pr(X_i | \text{parents}(X_i))$$

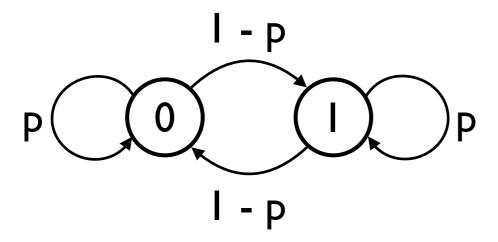
A Simple Example



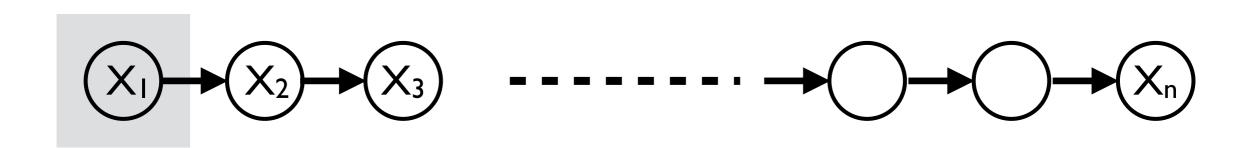
Model:

$$X_i$$
 in $\{0, 1\}$

State Transition Probabilities:



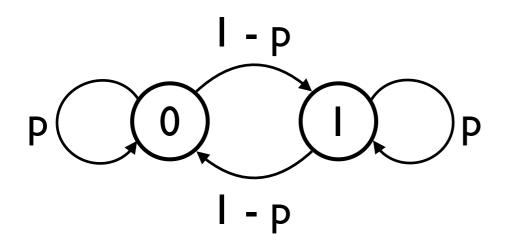
A Simple Example



Model:

$$X_i$$
 in $\{0, 1\}$

State Transition Probabilities: $Pr(X_2 = X_2)$

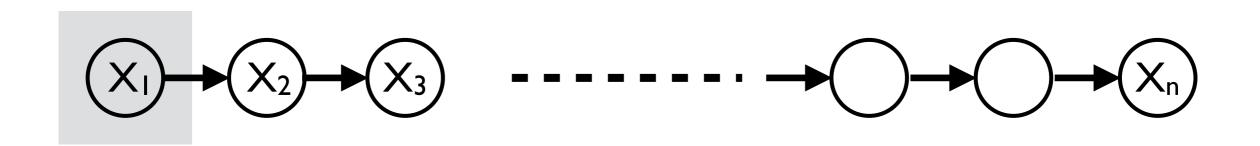


$$Pr(X_2 = 0 | X_1 = 0) = p$$

 $Pr(X_2 = 0 | X_1 = 1) = 1 - p$

• • • •

A Simple Example



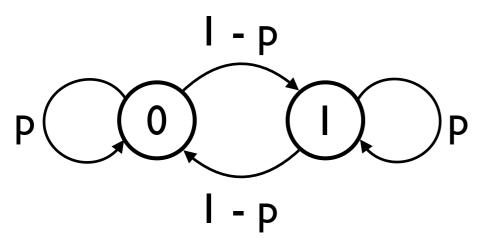
Model:

 X_i in $\{0, 1\}$

$Pr(X_2 = 0 | X_1 = 0) = p$

$$Pr(X_2 = 0 | X_1 = I) = I - p$$

State Transition Probabilities:



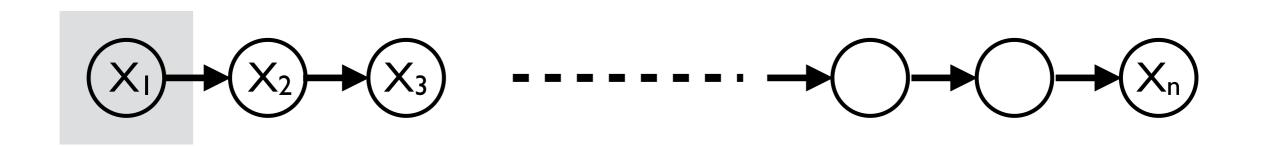
Pr(X_i = 0| X_I = 0) =
$$\frac{1}{2} + \frac{1}{2}(2p-1)^{i-1}$$

Pr(X_i = 0| X_I = I) = $\frac{1}{2} - \frac{1}{2}(2p-1)^{i-1}$

$$Pr(X_i = 0 | X_I = I) = \frac{1}{2} - \frac{1}{2}(2p-1)^{i-1}$$

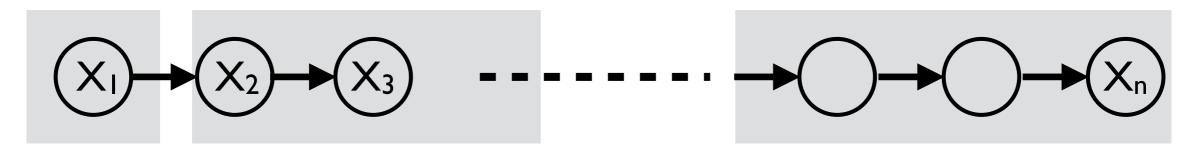
Influence of X₁ diminishes with distance

Algorithm: Main Idea



Goal: Protect X₁

Algorithm: Main Idea

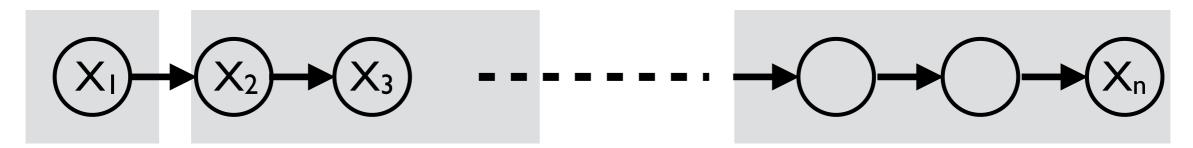


Local nodes (high correlation)

Rest (almost independent)

Goal: Protect X_I

Algorithm: Main Idea



Local nodes (high correlation)

Rest (almost independent)

Goal: Protect X_I

Add noise to hide local nodes

Small correction for rest

Measuring "Independence"

Max-influence of X_i on a set of nodes X_R :

$$e(X_R|X_i) = \max_{a,b} \sup_{\theta \in \Theta} \max_{x_R} \log \frac{\Pr(X_R = x_R|X_i = a, \theta)}{\Pr(X_R = x_R|X_i = b, \theta)}$$

Low $e(X_R|X_i)$ means X_R is almost independent of X_i

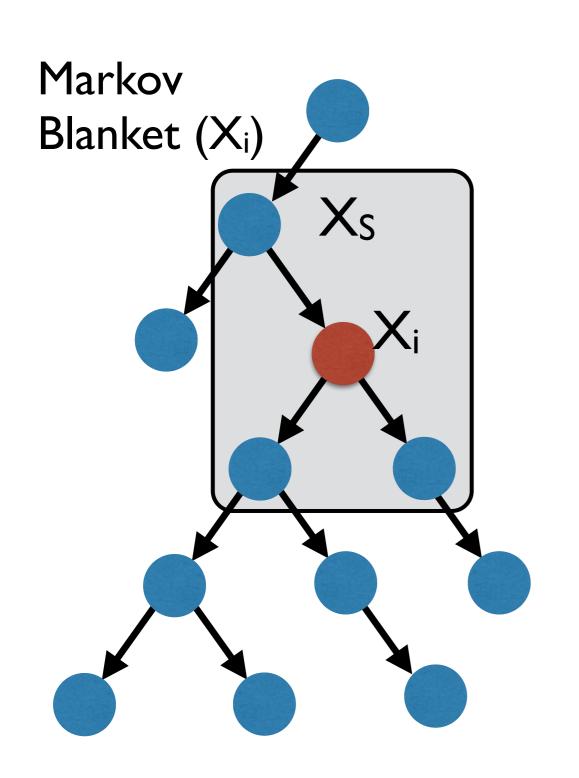
To protect X_i , correction term needed for X_R is $exp(e(X_R|X_i))$

How to find large "almost independent" sets

Brute force search is expensive

Use structural properties of the Bayesian network

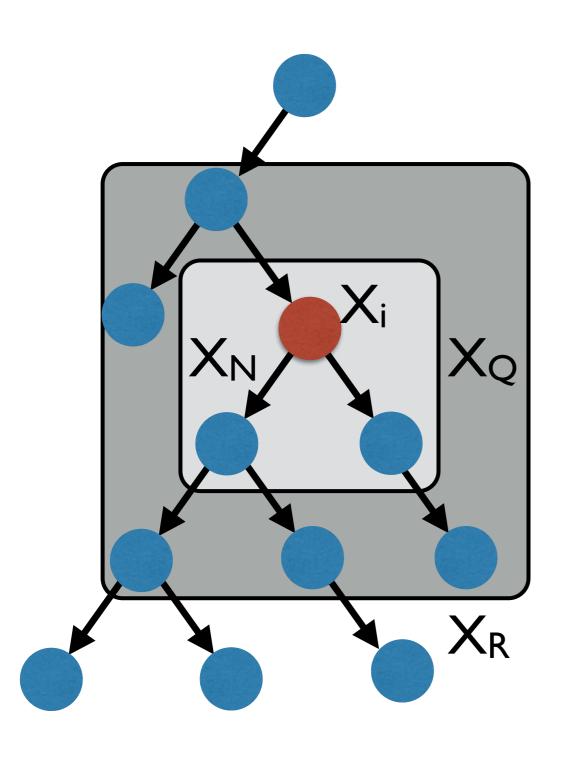
Markov Blanket



Markov Blanket(X_i) =
Set of nodes X_S s.t X_i is
independent of X\(X_i U X_S)
given X_S

(usually, parents, children, other parents of children)

Define: Markov Quilt

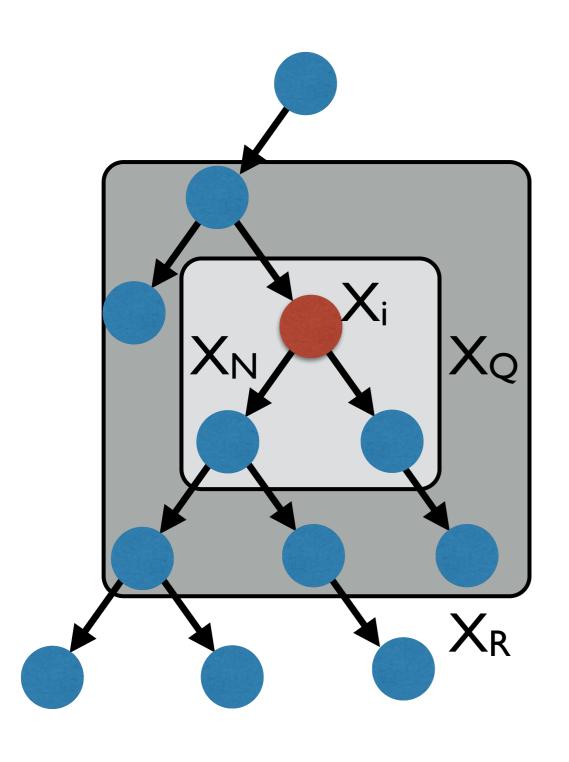


X_Q is a Markov Quilt of X_i if:

- I. Deleting X_Q breaks graph into X_N and X_R
- $2. X_i$ lies in X_N
- 3. X_R is independent of X_i given X_Q

(For Markov Blanket $X_N = X_i$)

Why do we need Markov Quilts?

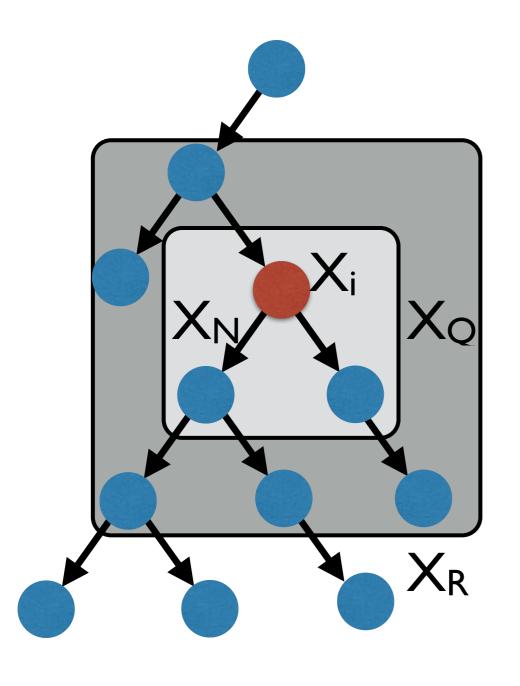


Given a Markov Quilt,

 $X_N = local nodes for X_i$

 $X_Q U X_R = rest$

From Markov Quilts to Amount of Noise



Let X_Q = Markov Quilt for X_i Stdev of noise to protect X_i :

Noise due to X_N

Score(X_Q) =
$$\frac{card(X_N)}{\epsilon - e(X_Q|X_i)}$$

Correction for $X_Q U X_R$

Search all Markov Quilts to find one that needs min noise

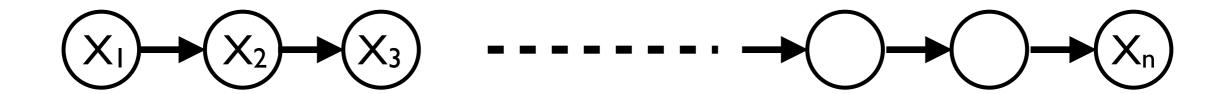
Privacy Properties

Privacy: MQM is ϵ -Pufferfish private

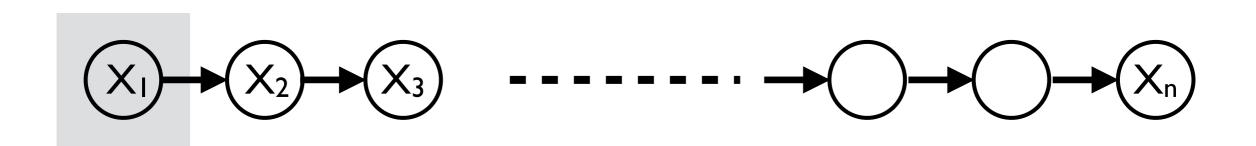
Graceful Composition

MQM for Markov Chains has:

- Additive sequential composition
- Parallel composition with a correction term



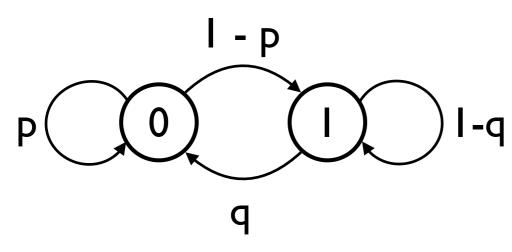
Simulations - Task



Model:

 X_i in $\{0, 1\}$

State Transition Probabilities:



Model Class:

$$\Theta = [\ell, 1 - \ell]$$

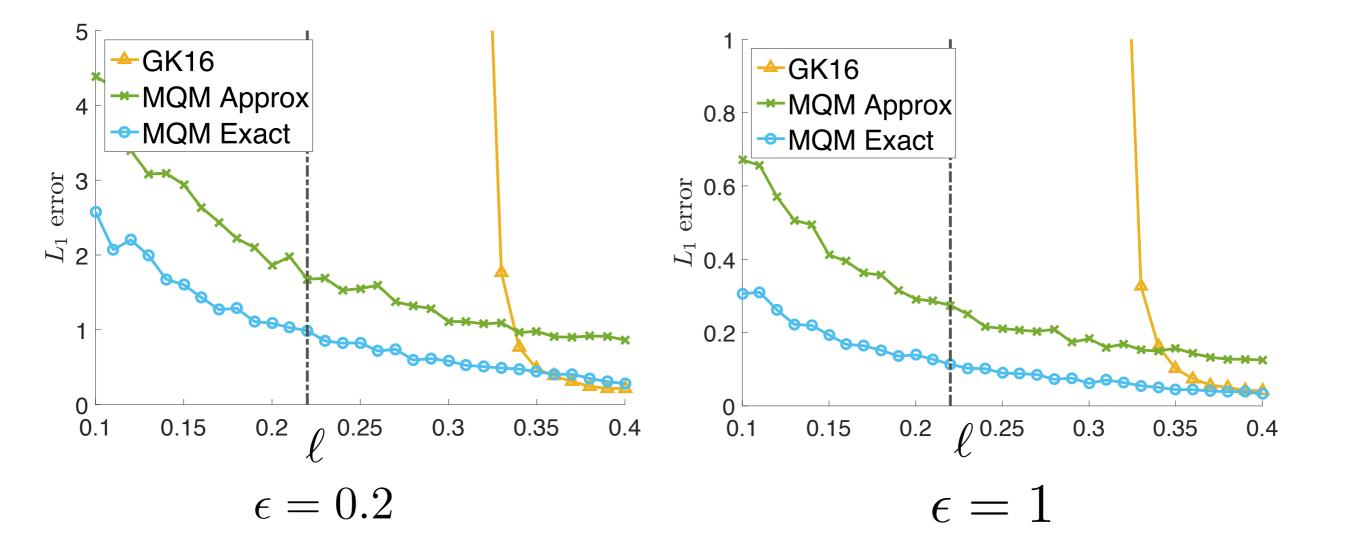
(implies p and q can lie anywhere in Θ)

Sequence length = 100

Simulations - Results

Methods:

- Two versions of Markov Quilt Mechanism (MQMExact, MQMApprox)
- **GK16**



Real Data - Activity Measurement

Dataset on physical activity by three groups of subjects: 40 cyclists, 16 older women and 36 overweight women

4 states (active, standing still, standing moving, sedentary)

Over 9,000 observations per subject

 $\Theta = \{ \text{ Empirical data generating distribution } \}$

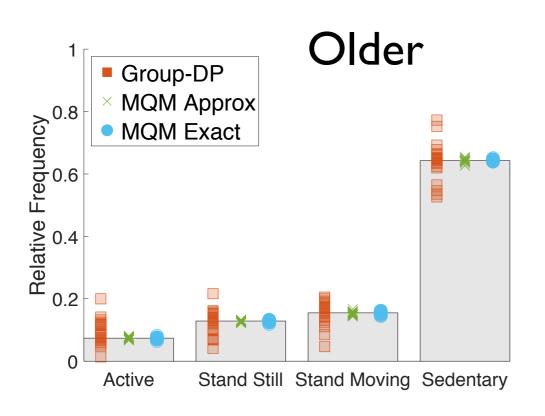
Methods:

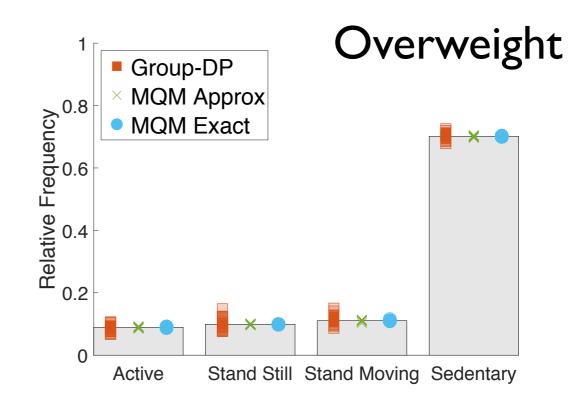
MQMExact and MQMApprox

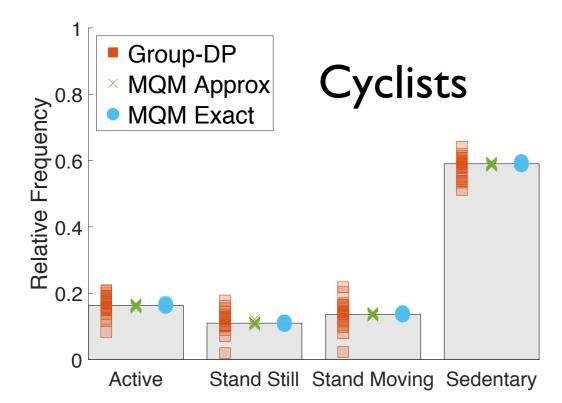
GroupDP

GK16 does not apply

Real Data - Activity Measurement







Aggregated results (over groups)

$$\epsilon = 1$$

Real Data - Power Consumption

Dataset on power consumption in a single household

Power consumption discretized to 51 levels (51 states)

Over I million observations

 $\Theta = \{ \text{ Empirical data generating distribution } \}$

Methods:

MQMExact vs. MQMApprox

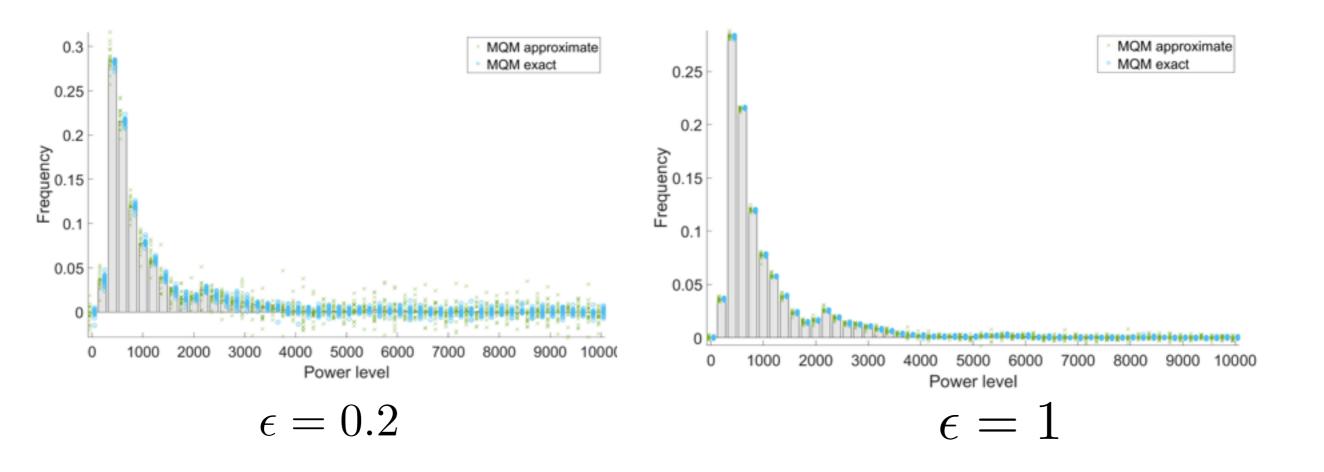
GK16 does not apply

GroupDP has too little utility

Real Data - Power Consumption

Methods:

Two versions of Markov Quilt Mechanism (MQMExact, MQMApprox)



Conclusion

- Real problems have complex privacy challenges
- Rigorous privacy definitions are available
- For any privacy problem, important to think:
 - What do we need to hide?
 - What do we need to reveal?

References

- "Differentially Private Continual Release of Graph Statistics", S. Song,
 S. Mehta, S. Vinterbo, S. Little and K. Chaudhuri, Arxiv, 2018
- "Pufferfish Privacy Mechanisms for Correlated Data", S. Song, Y.
 Wang and K. Chaudhuri, SIGMOD 2018.
- "Composition Properties of Inferential Privacy on Time-Series Data",
 S. Song and K. Chaudhuri, Allerton 2018.

Thanks!







